Threshold dynamics for the mean curvature motion of networks with unequal surface tensions.

Selim Esedoglu

Threshold dynamics is an algorithm for moving an interface (e.g.a surface in 3D) by mean curvature motion. It was proposed by Merriman,Bence, and Osher in 1992, and also extended to networks of surfaces in the same paper. This dynamics arises as gradient flow for the sum of the areas of the surfaces in the network, and plays a prominent role in materials science applications where it describes the motion of grain boundaries in polycrystals (such as most metals) under heat treatment.
Further extension of the algorithm to gradient flow for a weighted energy where the area of each surface in the network is weighted by possibly different constants (called surface tensions) is of great interest for applications, but has remained elusive. In fact, previous attempts at this turn out to be flawed, mainly due to the difficulty of ensuring that certain natural angle conditions are satisfied along triple curves (where three surfaces meet). We describe how to extend threshold dynamics to this unequal surface tension setting.
Joint work with Felix Otto.